

Hypotheses

Null vs Alternative Hypotheses

- **Null Hypothesis, H_0** , is the claim regarding the population statistic that we are examining.
 - ▷ It is stated using the following notation
 - H_0 : characteristic = value
- **Alternative Hypothesis, H_a** , is the competing claim; we are trying to find evidence for this.
 - ▷ It will have one of three forms:
 - H_a : characteristic < value
 - H_a : characteristic > value
 - H_a : characteristic \neq value

P-values and Proving H_a

- The **P-value** of a test result is the probability of that result occurring if H_0 is true
 - ▷ We compare this to an agreed-upon threshold (the **significance level, α**).

P-value < α – we have convincing evidence that H_a is true.

P-value $\geq \alpha$ – we have failed to disprove H_0 and we cannot take H_a as true.

Note that we have *not* proven H_0 ; we have simply been unable to disprove it.

Type 1 & Type 2 Errors; Power

- **Type 1 Error** - Reject H_0 when H_0 is actually true
 - ▷ *i.e.*, falsely accept H_a
 - The significance level, α , is the probability of getting a type 1 error.
- **Type 2 Error** - Not rejecting H_0 when H_0 is actually false
 - ▷ *i.e.*, Rejecting H_a when H_a is actually true
 - β is the probability of getting a type 2 error
- **Power** - The probability that a test will correctly confirm H_a when H_a is, in fact, true.
 - ▷ Power = $1 - \beta$

Testing a Population Proportion, π

Calculating a P-value

- We calculate the **P-value** of a test result by calculating the z-score of our result, compared to the hypothesized value, and then finding the probability associated with this z-score.
- The **test statistic** (z-value) of our test result is:

$$z = \frac{\text{statistic} - \text{parameter}}{\text{stddev of statistic}} = \frac{p - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Z - Z score/test statistic; p - proportion in sample
 p_0 - hypothesized proportion; n - sample size

- H_0 should be rejected if the P-value $\leq \alpha$
 - α is the significance level chosen for the test.

Obtaining P-value from z score

- Upper-tailed test** $H_a: \pi > h$ P-value = $1 - P(z)$
- Lower-tailed test** $H_a: \pi < h$ P-value = $P(z)$
- Two-tailed test** $H_a: \pi \neq h$ if $z > 0$, P-value = $2(1 - P(z))$
if $z < 0$, P-value = $2P(z)$

π - Test value from H_0 ; h - hypothesized value;
 z - Z score/test statistic; $P(z)$ - P-value from z table

Validity requirements

This equation is valid if:

- Random data
- 10% rule
 $n \leq 0.1N$
- Large counts
 $n \cdot \hat{p} \geq 10$
 $n(1 - \hat{p}) \geq 10$

Testing a Population Mean, μ

Calculating the test statistic

The test statistic is a t score when testing a population mean.

$$t = \frac{\text{statistic} - \text{parameter}}{\text{stddev of statistic}} = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

t - t-score/test statistic; \bar{x} - proportion in sample
 s_x - sample standard deviation; n - sample size

Calculator Note

On your graphing calculator, three functions are associated with testing hypotheses:

- 1-PropZTest** Proportion test
- T-Test** Mean (s_x)
- Z-Test** Mean (σ)

Validity requirements

This equation is valid if:

- Random data
- 10% rule
 $n \leq 0.1N$
- Large counts
 $n \geq 30$