Testing Hypotheses

Hypotheses

Null vs Alternative Hypotheses

- *Null Hypothesis*, H_0, is the claim regarding the population characteristic that is initially assumed to be true.
  - ▹ Think of it as the initial claim regarding the characteristic.
    For example, "The average mpg of the Ford Tortoise is 217 mpg."
  - ▹ This is of the form "characteristic = value"
    For example "average mpg = 217 mpg"
- *Alternative Hypothesis*, H_a, is the competing claim.
  - ▹ This hypothesis answers the question being examined.
  - ▹ It is of the form "characteristic < value", "characteristic > value", or "characteristic ≠ value"
    For example, if the concern is that the car doesn't live up to its claim, "average mpg < 217 mpg"

Proving H_a

- We settle the question of whether H_a is true by seeing if our study results are consistent with H_0.
  - ▹ If our study results are very unlikely to occur if H_0 is true, then we can reject H_0 and accept H_a as true.
  - ▹ On the other hand, if our study results are reasonably possible when H_0 is true, then we have failed to disprove H_0 and we cannot take H_a as true.

    Note that we have *not* proven H_0; we have simply been unable to disprove it.

Errors & Levels of Significance

There are two ways your results could be in error:

- *Type 1 Error* - (false negative) - Reject H_0 when H_0 is actually true
- *Type 2 Error* - (false positive) - Not rejecting H_0 when H_0 is actually false

- The ***Level of Significance, α,*** of a test result is the probability of a Type 1 error.
- β is the probability of a Type 2 error.
- Note that α and β have an inverse relationship
- In designing a test procedure, identify the largest acceptable *αgu* and design the procedure for this value. (This will minimize β.)

Large-sample Testing of a Population Proportion, π

P-Values

We reject H_0 (and therefore prove H_a) based on the probability that a test result would occur if H_0 were true.

- The *P-value* of a result is the probability of getting the observed test result if H_0 is true.
    - We obtain this by calculating the z-score of our result, compared to the hypothesized value, and then looking the P-value up in the z table.
- Let h be the hypothesized value for π.

$$z = \frac{p - h}{\sqrt{\frac{h(1-h)}{n}}}$$

Z - Z score of the difference; ***p*** *- proportion in sample*
h *- hypothesized proportion;* ***n*** *- sample size*

- H_0 should be rejected if the P-value $\leq \alpha$

Obtaining P-value from z score

Let h be the hypothesized value and $p(z)$ be the probability read from the z table:

- *Upper-tailed test* - $H_a: \pi > h$ P-value = $1 - p(z)$
- Lower-tailed test - $H_a: \pi < h$ P-value = $p(z)$
- Two-tailed test - $H_a: \pi \neq h$ if $z > 0$, P-value = $2(1 - p(z))$
  if $z < 0$, P-value = $2p(z)$

Steps in a Hypothesis-testing Analysis

1. Describe the population characteristic to be tested.
2. State H_0
3. State Ha
4. Select the significance level, α
5. Determine the test statistic (the z equation above with h replaced with the hypothesized value)
6. Determine whether the analysis meets the requirements (random sample and large sample size)
7. Calculate z from the test and hypothesized values
8. Determine the P-value
9. State the conclusion by comparing the P-value to α.