Probability

The Basics

The *probability* of an event *A* happening is

$$P(A) = \frac{\text{Number of } \textit{successful} \text{ outcomes}}{\text{Number of } \textit{possible} \text{ outcomes}}$$

or, paraphrasing:

$$P(A) = \frac{\text{Number of ways } A \text{ can occur}}{\text{Number of ways to get anything at all}}$$

- *e.g.*, the probability of rolling a die and getting a 3 is $\frac{1}{6}$.
  - ▹ There's one 1 and 6 possible numbers.
- *e.g.*, the probability of picking a 2 from a deck of cards is $\frac{4}{52} = \frac{1}{13}$.
  - ▹ There are four 2's and 52 cards total.

Combining Probabilities

Given two events *A* and *B*:

Independent One event does not affect the outcome of the other.

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B \mid A)$$

▹ *e.g.*, the chance of rolling a die twice and getting a 3 both times is

$$P(A)\,P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Mutually Exclusive / Disjoint

Two outcomes that can never occur simultaneously; $P(A \cap B) = 0$.

$$P(A \cup B) = P(A) + P(B)$$

▹ *e.g.*, the chance of rolling a die getting either a 2 or a 3 is

$$P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

Not Mutually Exclusive

The events overlap; a single subject could fulfill both conditions.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

▹ *e.g.*, the chance of picking a card from a deck and getting either a 2 or a spade is

$$\frac{4}{52} + \frac{13}{52} - \frac{4}{52} \times \frac{13}{52} = \frac{4}{13}$$

Conditional (Dependent)

If A and B can happen simultaneously if B *has* happened, what is the probability that A will happen also?

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

This is read "The probability of B given A."

- *e.g.*, if 90% of the class passed a quiz and 60% of the class passed the quiz and the following test, what is the probability that a student who passed the quiz also passed the test?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.6}{.9} = 67\%$$

Independent Events

The occurance of A doesn't affect P(B) and vice versa. That is,

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

Complement of an event

The probability that an event *won't* happen.

Denoted variously by A^C, A', and $\overline{A}$

$$P(A^C) = 1 - P(A)$$

"At least one…"

Given a series of n trials in which the probablility of event A occuring in any one trial is P(A), the probability that *at least* one trial will have an occurance of A is

$$1 - (P(A^C))^n$$

- *e.g.*, if there is a .6 chance that any one stoplight will be red, what is the probability that you could pass through 8 stoplights and at least one will be red?

$$(1 - (.4)^8) = 0.99934$$