Confidence Intervals

A *confidence interval* (CI) is a range of values that have a stated probability ("*degree of confidence*") of containing the actual value of a population characteristic.

Population Proportions

The CI calculation is based on a simple proportion, p, of the successes within a sample population.

$$CI = p \pm Z_{crit}\sqrt{\frac{p(1-p)}{n}}$$

Note that the radical is an estimate of the standard deviation of the population.

Z_{crit} - Z score of the degree of confidence (also Z*);
p - proportion of successes; n - sample size

Z_{crit} (Z*) Values	
Confidence	*Z*
90%	1.645
95%	1.96
98%	2.33
99%	2.58

Restrictions

For the CI calculation to be valid, two conditions must be met for the sample:

- The sample must have been random; there were no interactions between the individual scores
- n must be large; specifically, $np \geq 10$ and $n(1-p) \geq 10$

CI for Other Population Characteristics

The CI calculation is based on a point estimate of the characteristic.

$$CI = (\text{point estimate of statistic}) \pm Z_{crit}\,(\text{standard deviation of the statistic})$$

Standard Error/Margin of Error

The *Standard Error* (SE) is the estimated standard deviation of the statistic.

The *Margin of Error* (ME) is the half-width of the confidence interval.

$$ME = Z_{crit}\sqrt{\frac{p(1-p)}{n}}$$

This is also the calculation for B, below.

The *bound on error of estimation*, *B*, is a margin of error desired to within a particular confidence level. This is a "target" ME for a particular confidence level. Calculated identically to ME, above.

Choosing Sample Size

The sample size needed to achieve a particular margin of error to a particular degree of confidence is:

$$n = .25\left(\frac{Z_{crit}}{ME}\right)^2$$

ME, here, is functionally the same as *B*.

Confidence Interval for Population Mean, μ

These techniques take the mean, $\bar{x}$, of sample data.

When σ is known

$$CI = \bar{x} \pm Z_{crit}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Z_{crit} - Z score of the degree of confidence; σ - population standard deviation; n - sample size

Requirements:

- x is the sample mean from a random sample
- The sample size, n ≥ 30

When σ is not known

$$CI = \bar{x} \pm t_{crit}\left(\frac{s}{\sqrt{n}}\right)$$

t_{crit} - t score of the degree of confidence and degrees of freedom; s - sample standard deviation; n - sample size

Requirements:

- x is the sample mean from a random sample
- The sample size, n ≥ 30