Dot and Cross Products

Dot Product

Converts two vectors to a scalar value.

- $\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\|\,\|\mathbf{B}\| \cos(\theta)$ ← θ is the angle between the two vectors
- $\mathbf{A} \cdot \mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$ ← A_1, A_1, etc. are the **i**, **j**, **k** coefficients for the two vectors

Cross Product

Results in a new vector.

- *Magnitude:* $\mathbf{A} \times \mathbf{B} = \|\mathbf{A}\|\,\|\mathbf{B}\| \sin(\theta)$
- *Direction:* perpendicular to the two vectors using the right-hand rule for $\mathbf{A} \to \mathbf{B}$

Notation

This document uses standard vector notation:

A **Heavy text** indicates a vector

$\|\mathbf{A}\|$ Double vertical bars denote the *length* of a vector.

Formal Definition of cross product

Given vectors

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$
$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

Note this is the determinant of the matrix

- $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Other Vector Calculations

Vector between two points

Given points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$,

- Vector $\mathbf{AB} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$

Unit Vector

Given $\mathbf{A} = (x, y, z)$, the unit vector parallel to **A** is

- $$\hat{\mathbf{a}} = \frac{x}{\|\mathbf{A}\|}\mathbf{i} + \frac{y}{\|\mathbf{A}\|}\mathbf{j} + \frac{z}{\|\mathbf{A}\|}\mathbf{k}$$

Projections

Projection of a onto b

This is the component of **a** in the direction of **b**

$$\mathrm{proj}_{b}a = \frac{a \cdot b}{\|b\|^2} \times b$$