

Vector Notation

In printed material, vector names are indicated by heavy text: **A**.

In handwriting, vectors are usually indicated by an arrow above the name: \vec{A}

$\mathbf{A} = \langle x, y \rangle$ Vector with specified x - and y -components

$\mathbf{B} = x\mathbf{i} + y\mathbf{j}$ Vector with specified unit-vector components

$\|\mathbf{A}\|$ or $[\mathbf{A}]$ Magnitude (*i.e.*, length) of vector **A**.

Basic Operations

In the following, $\mathbf{A} = \langle a_1, a_2 \rangle$, $\mathbf{B} = \langle b_1, b_2 \rangle$

Addition, subtraction, scalar multiplication

$$\mathbf{A} + \mathbf{B} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

$$c\mathbf{A} = \langle ca_1, ca_2 \rangle$$

c is a scalar constant
(*a.k.a.*, a number).

$$\mathbf{A} - \mathbf{B} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

Dot Product

The dot product of two vectors is a scalar value, *i.e.*, a number, not a vector.

- $\mathbf{A} \cdot \mathbf{B} = a_1b_1 + a_2b_2$
- $\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \|\mathbf{B}\| \cos(\theta)$ ← θ is the angle between the two vectors

Perpendicular

If **A** and **B** are perpendicular to each other, then

$$\mathbf{A} \cdot \mathbf{B} = 0$$

Other Vector Calculations

Vector between two points

Given points $A(x_1, y_1)$ and $B(x_2, y_2)$,

- Vector $\mathbf{AB} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$

Unit Vector

$\hat{\mathbf{a}}$, the unit vector parallel to $\mathbf{A} = \langle a_1, a_2 \rangle$, is

- $\hat{\mathbf{a}} = \frac{x}{\|\mathbf{A}\|}\mathbf{i} + \frac{y}{\|\mathbf{A}\|}\mathbf{j}$

Projection of a onto b

This is the component of **a** in the direction of **b**

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \times \mathbf{b}$$

Cross Product

The cross product results in a new vector that is perpendicular to the original vectors. Note that the cross product is applicable only to three-dimensional vectors.

- *Magnitude:* $\mathbf{A} \times \mathbf{B} = \|\mathbf{A}\| \|\mathbf{B}\| \sin(\theta)$
- *Direction:* perpendicular to the original vectors using the right-hand rule for $\mathbf{A} \rightarrow \mathbf{B}$

Formal Definition of cross product

Given vectors

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$

$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

Note this is the determinant of the matrix

$$\bullet \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

