Conic Sections in Polar Coordinates

Conversions

Rectangular → Polar

$x = r\cos(\theta)$

$y = r\sin(\theta)$

Polar → Rectangular

$r^2 = x^2 + y^2$

$\tan(\theta) = \frac{y}{x}$

Conic Sections

Alternative Definition of a Conic Section

The set of points whose distance from a fixed point (the *focus*) and distance to a fixed line (the *directrix*) is a constant ratio.

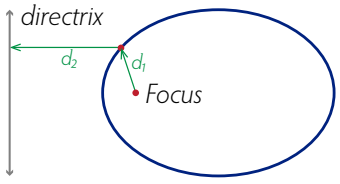
- The constant ratio is the *eccentricity* (e) of the curve; its value determines the type of conic.
  - $0 < e < 1$ Ellipse
  - $e = 1$ Parabola
  - $e > 1$ Hyperbola

Eccentricity and Conic Type

In the diagrams below, $e = \frac{d_1}{d_2}$

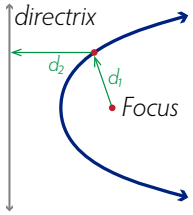
$0 < e < 1$

Ellipse



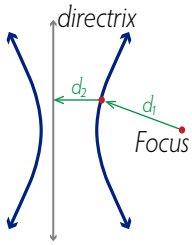
$e = 1$

Parabola



$e > 1$

Hyperbola



Polar Equations of Conic Sections

Vertical Directrix (*symmetric about polar axis*)

$$r = \frac{ep}{1 \pm e\cos\theta}$$

Horizontal Directrix (*symmetric about* $\theta = \frac{\pi}{2}$)

$$r = \frac{ep}{1 \pm e\sin\theta}$$

e = eccentricity; $|p|$ = distance between focus and directrix

Rev.