

Basic Surfaces

The following assume the center of the surface is at the origin. If the center is at (x_0, y_0, z_0) , replace $x, y,$ and $z,$ below, with $(x - x_0), (y - y_0),$ and $(z - z_0).$

Sphere

$$x^2 + y^2 + z^2 = r^2$$

Plane

$$ax + by + cz + d = 0$$

- $\langle a, b, c \rangle$ is the plane's normal vector.

Cylinder

Any equation of two variables. e.g., $x^2 + y^2 = 4$

- The equation is the generating curve (directrix)
- The rulings are parallel to the missing axis.

Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- $a, b,$ and c are the radii along the corresponding axis.

Hyperboloid, 1-sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

- The axis corresponds to the negative variable.

Hyperboloid, 2-sheet

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

- The axis corresponds to the positive variable.

Determining a Trace

To determine the nature of the trace of a surface on one of the coordinate planes, set the other variable to 0 and see what kind of conic remains.

Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

- The axis corresponds to the negative variable.

Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- The axis corresponds to the linear variable.

Hyperbolic Paraboloid

$$z = \frac{y^2}{a^2} - \frac{x^2}{b^2}$$

- Saddle opens down along the linear variable.

Surface of Revolution

A surface of revolution is defined by two items:

- A two-dimensional **generating curve** (e.g., $y = x^2$)
- An **axis of rotation** (e.g., the x-axis)

The axis of rotation must be one of the axes taking part in the generating curve.

Equations

The equation of the surface is determined by the axis of rotation:

- x-axis $y^2 + z^2 = [r(x)]^2$ ← $r(x)$ is the generating curve
- y-axis $x^2 + z^2 = [r(y)]^2$
- z-axis $x^2 + y^2 = [r(z)]^2$