Newton's Method

Description

Newton's Method is a process for finding an approximate solution to the function

$$f(x) = 0.$$

The process is iterative; that is, it operates by finding successive approximations to the solution, each approximation closer to the real solution than the last, until you find an approximation that is "close enough."

- "Close enough" usually means the successive approximations are no longer differing from each other by a meaningful amount.
- *e.g.*, perhaps each new approximation is only .0001 different from the previous one.

The Short Version *(use this)*

- Newton's method starts with an initial guess you must make as to the *x*-value of the solution.
  - *The actual value of the guess doesn't matter*; it can be as accurate or wildly wrong as you wish, though the closer you are to the real value, the fewer times you'll need to go through the loop.
- Successive approximations can be expressed as a recursive sequence, as follows:

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

Your initial x_n will be the guess that you make at the start of the process.

The Method

The above recursive equation is the result of the following process:

1 Find the derivative of $f(x)$.

2 Make a guess as to what the solution value might be; call this number x_0.

The loop:

3 Calculate the equation of the tangent line to the function at x_0.

- Use $f'(x)$ for the slope and $(x_0, f(x_0))$ for the point.

4 Calculate the x-intercept of the tangent line.

- *i.e.*, set the equation equal to zero and solve for *x*.

5 Use the intercept for your new x_0 and go back to Step 3.

Repeat Steps 3, 4, and 5 until x_0 is "close enough."