ε-δ Proof of Limits

for Linear Equations

Epsilon-Delta Definition of Limit

$\lim_{x \to a} f(x) = L$ is true if, for every $\varepsilon > 0$ there exists $\delta > 0$ such that, for all x,

if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$

In other words, if you want $f(x)$ to be within a particular very small distance (ε) of L, then however small ε may be, there will be a value of x within a small distance (δ) of a, that will make it so.

How to Prove a Limit

Problem: Prove $\lim_{x \to 2} (7x - 4) = 10$

Step 1: The preliminary: Determine δ in terms of ε

1 Start with the definition as it applies to our specific function and limit.

$$|7x - 4 - 10| < \varepsilon \qquad 0 < |x - 2| < \delta$$

2 Reduce the *epsilon* equation

$$|7x - 14| < \varepsilon$$

$$7|x - 2| < \varepsilon$$

3 Rearrange the inequality so $|x - 2|$ is on one side.

$$|x - 2| < \frac{\varepsilon}{7}$$

$\delta = \frac{\varepsilon}{7}$ ⟵ Because $|x - 2| < \delta$; this will be our starting point for step 2

Step 2: The Proof of $\lim_{x \to 2}(7x - 4) = 10$ (in which we run step 1 backward)

To prove the limit is correct, we need to show that if we have an x that is within a particular small distance (δ) of 2 (that is, $|x - 2| < \delta$), then $f(x)$ will be within a *predictable* distance (ε) of 10 (that is, $|f(x) - 10| < \varepsilon$).

Let $\delta = \frac{\varepsilon}{7}$	We did all of step 1 in order to determine how δ and ε are related
Let $\lvert x - 2\rvert < \delta$	'Cause this is the meaning of δ
$\lvert x - 2\rvert < \frac{\varepsilon}{7}$	Substitution
$7\lvert x - 2\rvert < \varepsilon$	Now let's solve for ε
$\lvert 7x - 14\rvert < \varepsilon$	And massage the left-hand side to match our $f(x) - 10$
$\lvert 7x - 4 - 10\rvert < \varepsilon$	
$\lvert f(x) - 10\rvert < \varepsilon$	Substitution
$\therefore \lim_{x \to 2} (7x - 4) = 10$	Because the two conditions are met: $\lvert x - 2\rvert < \delta$ and $\lvert f(x) - 10\rvert < \varepsilon$

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