# Tangent Planes and Normal Lines

## Definition

Given $F(x, y, z) = 0$ and point $P(x_0, y_0, z_0)$ such that $\nabla F(x_0, y_0, z_0) \neq 0$

- *Tangent plane* The plane that passes through $P$ and is normal to $\nabla F(x_0, y_0, z_0)$.
- *Normal Line* The line through $P$ in the directon of $\nabla F(x_0, y_0, z_0)$.

## Equations

- *Plane* tangent to $F(x, y, z) = 0$ at $(x_0, y_0, z_0)$
  - ▹ $F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$
- *Line* normal to $F(x, y, z) = 0$ at $(x_0, y_0, z_0)$
  - ▹ Gradient: $\nabla F(x_0, y_0, z_0)$

## Angle of Inclination of a Plane

Angle between the plane $F(x, y, z) = 0$ and the $xy$-plane:

$$\cos\theta = \frac{|\mathbf{n} \cdot \mathbf{k}|}{\|\mathbf{n}\|} \qquad 0 \leq \theta \leq \pi/2$$

*$\mathbf{n}$ - normal vector; $\mathbf{k}$ - z-vector; i.e., < 0, 0, 1 >*