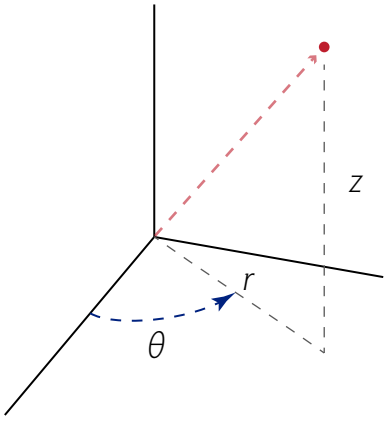
Cylindrical & Spherical Coordinates

Cylindrical Coordinates

- Positions are specified in terms of an angle, θ, and distance (r) from the *pole* (the origin) in the x-y plane and a distance, z, above the x-y plane, as at right.
- Positions are expressed as an ordered triplet, (r, θ, z).

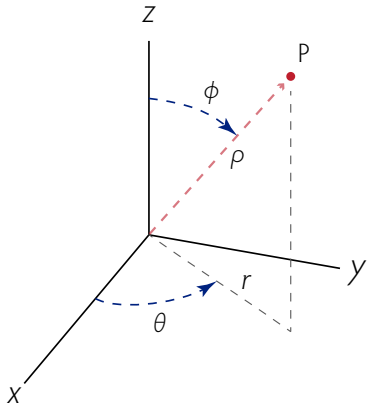


Conversions

- Cylindrical (r, θ, z) to Rectangular (x, y, z)
  - ▷ $x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad z = z$
- Rectangular (x, y, z) to Cylindrical (r, θ, z)
  - ▷ $r^2 = x^2 + y^2$ $\quad \tan\theta = \frac{y}{x}$ $\quad z = z$

Spherical Coordinates

- Positions are specified in terms of an angle θ and distance ρ from the pole (the origin) in the x-y plane and an angle ϕ from the z axis, as at right.
- Positions are expressed as an ordered triplet, (r, θ, ϕ).



Conversions

Remember that ρ is the distance from the pole (not an angle).

Spherical ⟷ Rectangular

- Spherical (ρ, θ, ϕ) to Rectangular (x, y, z)
  - ▷ $x = \rho\sin\phi\cos\theta$ $\quad y = \rho\sin\phi\sin\theta$ $\quad z = z$
- Rectangular (x, y, z) to Spherical (ρ, θ, ϕ)
  - ▷ $x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad z = z$

Spherical ⟷ Cylindrical

- Spherical (r, θ, ϕ) to Cylindrical (r, θ, z)
  - ▷ $r^2 = \rho^2\sin^2\phi$ $\quad \theta = \theta$ $\quad z = \rho\cos\phi$
- Cylindrical (r, θ, ϕ) to Spherical (r, θ, z)
  - ▷ $\rho = \sqrt{r^2 + z^2}$ $\quad \theta = \theta$ $\quad \phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2 - a^2}}\right)$