Lines and Planes in Space

Lines

Equation of a Line in Space

For a line parallel to the vector $<a, b, c>$ (the line's **direction vector**) and passing through point (x_0, y_0, z_0)

- *Parametric equations*

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

a, b, and c are the **direction numbers** of the line.

- *Symmetric equations*

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Distance Between a Point and a Line in Space

Distance between point Q and a line in space.

$$D = \frac{\| \overrightarrow{PQ} \times \mathbf{u} \|}{\|\mathbf{u}\|}$$

P - Any point on the line; $\mathbf{u}$ - line's direction vector

Planes

Equation of a Plane

The equation of a plane containing the point (x, y, z) and having normal vector $<a, b, c>$:

Standard Form: $a(x - x_1) + b(y - y_1) + c(z - z_1)$

General Form: $ax + by + cz + d = 0$

Distance Between a Point and a Plane

The distance between a point Q and a plane:

$$D = \| \text{proj}_{\mathbf{n}} \overrightarrow{PQ} \| = \frac{| \overrightarrow{PQ} \cdot \mathbf{n} |}{\|\mathbf{n}\|}$$

P - Any point in the plane; $\mathbf{n}$ - the plane's normal vector

Alternative Equation

Distance between a point (x_0, y_0, z_0) and plane $ax + by + cz + d = 0$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Line of Intersection

To find the direction numbers of the line of intersection of two planes:

- Take the cross product of the two normal vectors.
- Simultaneously solve the two planes' equations:

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0$$

Angle Between Two Planes

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\| \, \|\mathbf{n}_2\|}$$