

## Trig Substitution

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### The Relevant Trig Identities

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

### The Substitutions

$$\int \sqrt{a^2 - u^2} \, du$$

Substitution: let  $u = a \sin \theta$

Result:  $\sqrt{a^2 - u^2}$  becomes  $a \cos \theta$

$$\int \sqrt{a^2 + u^2} \, du$$

Substitution: let  $u = a \tan \theta$

Result:  $\sqrt{a^2 + u^2}$  becomes  $a \sec \theta$

$$\int \sqrt{u^2 - a^2} \, du$$

Substitution: let  $u = a \sec \theta$

Result:

$\sqrt{u^2 - a^2}$  becomes:  $a \tan \theta$  for  $u > a$

$-a \tan \theta$  for  $u < -a$

### Theorems ( $a > 0$ )

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$$\int \sqrt{a^2 - u^2} \, du$$

$$\int \sqrt{a^2 - u^2} \, du = \frac{1}{2} \left( a^2 \arcsin \frac{u}{a} + u \sqrt{a^2 - u^2} \right) + C$$

$$\int \sqrt{u^2 - a^2} \, du$$

$$\int \sqrt{u^2 - a^2} \, du = \frac{1}{2} \left( u \sqrt{u^2 - a^2} - a^2 \ln |u + \sqrt{u^2 - a^2}| \right) + C \quad u > a$$

$$\int \sqrt{u^2 + a^2} \, du$$

$$\int \sqrt{u^2 + a^2} \, du = \frac{1}{2} \left( u \sqrt{u^2 + a^2} + a^2 \ln |u + \sqrt{u^2 + a^2}| \right) + C$$