# Trigonometric Integrals

## Trig Functions with Exponents

Following are techniques for evaluating integrals of the form

$$\int \sin^m x \cos^n x \, dx \quad \text{and} \quad \int \sec^m x \tan^n x \, dx$$

where *m* and *n* are positive integers.

### Three Cases

- Power of sine is odd
  - Save one sine factor for the *du*, convert the remaining factors to cosines, then expand and integrate.

    $$\int \sin^{2k+1} x \cos^n x \, dx \;\rightarrow\; \int (\sin^2 x)^k \cos^n x \sin x \, dx \;\rightarrow\; \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx$$
  - Substitution: $u = \cos x$
- Power of cosine is odd
  - Save one cosine factor for the *du*, convert the remaining factors to sines, then expand and integrate.

    $$\int \sin^m x \cos^{2k+1} x \, dx \;\rightarrow\; \int \sin^m x \, (\cos^2 x)^k \cos x \, dx \;\rightarrow\; \int \sin^m x \, (1 - \sin^2 x)^k \cos x \, dx$$
  - Substitution: $u = \sin x$
- Powers of sine and cosine are both even
  - Repeatedly use the identities

    $$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

    until you get an odd power of cosine, then apply the second technique, above.