Power & p-Series

p- and Harmonic Series

- A *p-series* is a series of the form

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n^p} \qquad \text{where } p > 0$$

For example,

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \tfrac{1}{4} + \tfrac{1}{9} + \tfrac{1}{16} + \ldots$$

p-Series Convergence

A p-series converges if $p > 1$ and diverges if $0 < p \leq 1$.

- A *harmonic series* is a p-series with $p = 1$, *i.e.,*

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \ldots$$

- The *general form* for a harmonic series is

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{an + b} \qquad \text{(note that } p \text{ is still equal to 1)}$$

Power Series

- A *power series* is a series of the form

$$F(x) = \sum_{n=0}^{\infty} a_n(x - c)^n$$

This is referred to as a "power series centered at c."

where c is a constant and a_n is a coefficient derived from n

Note that if a_n is a constant and $c = 0$, a power function becomes a classic geometric series.

Interval of convergence

- The *interval of convergence* is the set of all x for which the power series converges.
- The *radius of convergence* is ½ the size of the interval of convergence.
- Three possibilities:
  - ▹ Series converges only when $x = c$
  - ▹ Series converges when $|x - c| < R$, where R, the radius of convergence, is real and positive
  - ▹ Series converges for a values of x
- To find the interval of convergence, solve the ratio test for $L < 1$.
  - ▹ Must test the endpoints of the interval of convergence separately.

Remember the Ratio Test?

Calculate

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$L < 1$	Series converges
$L > 1$	Series diverges
$L = 1$	Inconclusive