Volumes & Surfaces

Volumes of Revolution

Disk Method

Horizontal Axis of Revolution

$$V = \pi \int_a^b [r(x)]^2 \, dx$$

Vertical Axis of Revolution

$$V = \pi \int_a^b [r(y)]^2 \, dy$$

Washer Method

Horizontal Axis of Revolution

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx$$

Vertical Axis of Revolution

$$V = \pi \int_a^b \left([R(y)]^2 - [r(y)]^2\right) dy$$

Cylindrical Shell Method

Horizontal Axis of Revolution

$$V = 2\pi \int_a^b y f(y) \, dy$$

Vertical Axis of Revolution

$$V = 2\pi \int_a^b x f(x) \, dx$$

Arc Length, *s*

y = f(x)

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

x = g(y)

$$s = \int_a^b \sqrt{1 + [g'(y)]^2} \, dy$$

Area of Surface of Revolution

Horizontal Axis, y = f(x), axis y = c

$$A = 2\pi \int_a^b (f(x) - c)\sqrt{1 + [f'(x)]^2} \, dx$$

Vertical Axis, x = g(y), axis x = c

$$A = 2\pi \int_a^b (g(y) - c)\sqrt{1 + [g'(y)]^2} \, dx$$