Trigonometric Substitutions

The Relevant Trig Identities

$\cos^2\theta = 1 - \sin^2\theta$ $\quad$ $\sec^2\theta = 1 + \tan^2\theta$ $\quad$ $\tan^2\theta = \sec^2\theta + 1$

The Substitutions

$\int \sqrt{a^2 - u^2}$

Substitution: let $u = a\sin\theta$

Result: $\sqrt{a^2 - u^2}$ becomes $a\cos\theta$

$\int \sqrt{a^2 + u^2}$

Substitution: let $u = a\tan\theta$

Result: $\sqrt{a^2 + u^2}$ becomes $a\sec\theta$

$\int \sqrt{u^2 - a^2}$

Substitution: let $u = a\sec\theta$

Result: $\sqrt{u^2 - a^2}$ becomes: $a\tan\theta$ for $u > a$

$-a\tan\theta$ for $u < -a$