Definitions and Theorems

Definitions

Continuous

A function is *continuous* at h if

$$\lim_{x \to h} f(x) = f(h)$$

Differentiable

A function is *differentiable* on open interval (a,b) if $f'(x)$ exists for every value of x on (a,b)

- $f(x)$ needs to be continuous at x for it to be differentiable at x.
- $f(x)$ can't be a cusp or asymptotic at x.
- A function is differentiable at $x = c$ if df/dx to the left of c is equal to df/dx to the right.
  - *i.e.*, the slopes coming from the left and right must be the same.

Critical Numbers

If $f(x)$ is defined at $x = c$, then c is a *critical number* of f if one of the following is true:

- $f'(c) = 0$
  - *i.e.*, $f(c)$ is a local maximum or minimum
- $f(x)$ is not differentiable at c.
  - *i.e.*, $f(c)$ is a discontinuity or a cusp.

Theorems

Intermediate Value Theorem

If f is continuous on $[a,b]$ and k is between $f(a)$ and $f(b)$, then there exists a value c on $[a,b]$ such that

$$f(c) = k$$

Mean Value Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b), then there exists a value c on $[a,b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- *In other words,* there will be some value of x between a and b where the instantaneous slope of the function is equal to the average slope between a and b.

Rolle's Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b), and $f(a) = f(b)$, then there exists a value c on $[a,b]$ such that

$$f'(c) = 0$$