Polar Coordinates

Conversions

Rectangular → Polar

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

Polar → Rectangular

$$r^2 = x^2 + y^2$$
$$\tan(\theta) = \frac{y}{x}$$

Derivatives, Tangent Lines, etc.

Slope

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Note that the tangent is horizontal when $dy/d\theta$ is zero and vertical when $dx/d\theta$ is zero

Tangent Lines at the Pole

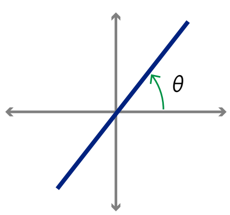
If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the curve $r = f(\theta)$

Common Curves

Lines

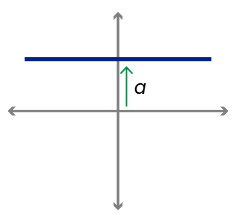
θ = constant

Line through origin



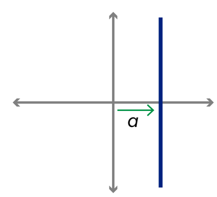
$r = a\csc(\theta)$

Horizontal line



$r = a\sec(\theta)$

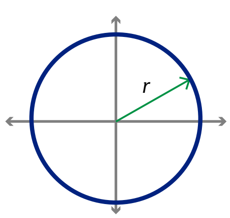
Vertical line



Circles

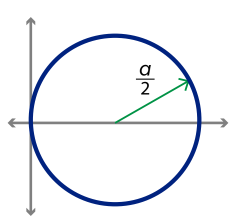
r = constant

Circle centered at origin



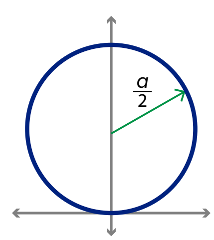
$r = a\cos(\theta)$

Circle centered at $(\frac{a}{2}, 0)$



$r = a\sin(\theta)$

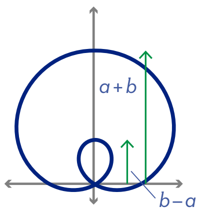
Circle centered at $(0, \frac{a}{2})$



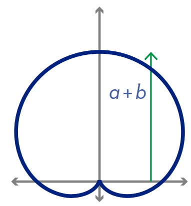
Limaçon $r = a \pm b\sin(\theta)$

Cosine limaçons extend to the left or right, rather than up or down.

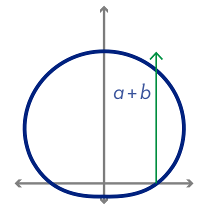
$a < b$



$a = b$



$a > b$

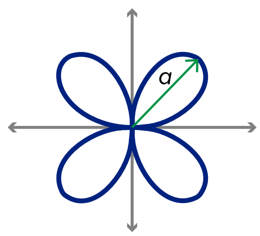


Rose

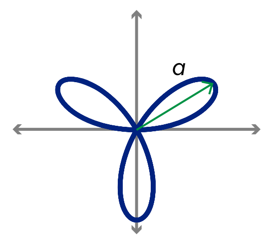
$r = a\sin(n\theta)$

- *n even*: $2n$ petals
- *n odd*: n petals
- 1st petal starts tangent to the axis

$r = a\sin(2\theta)$



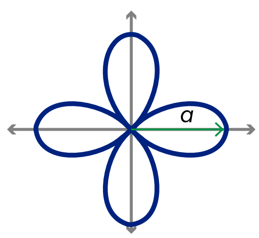
$r = a\sin(3\theta)$



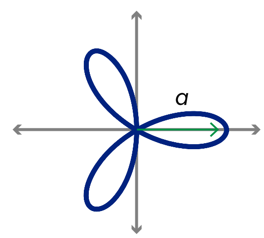
$r = a\cos(n\theta)$

- *n even*: $2n$ petals
- *n odd*: n petals
- 1st petal starts centered on the axis

$r = a\cos(2\theta)$



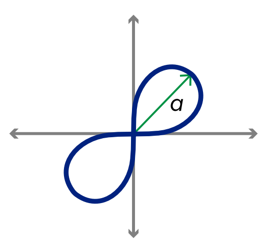
$r = a\cos(3\theta)$



Lemniscates

- *sine*: petals aligned 45°
- *cosine*: petals on x-axis

$r^2 = a^2\sin(2\theta)$



$r^2 = a^2\cos(2\theta)$