Polar Coordinates: Area, Arc Length

Area in Polar Coordinates

For a function r = $f(\theta)$, the area bounded by the arc and the radial lines $\theta = \alpha$ and $\theta = \beta$ is

$$A = ½\int_{\alpha}^{\beta}[f(\theta)]^2 d\theta = ½\int_{\alpha}^{\beta} r^2 d\theta$$

Arc Length of a Polar Curve

For a function r = $f(\theta)$, the length of the arc encompassed by the radial lines $\theta = \alpha$ and $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta}\sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta}\sqrt{r^2 + [dr/d\theta]^2}\, d\theta$$

Area of a Surface of Revolution

The area of the surface formed by revolving the graph of r = $f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows.

About the Polar Axis (θ = 0)

$$S = 2\pi\int_{\alpha}^{\beta} f(\theta)\sin(\theta)\sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta} f(\theta)\sin(\theta)\sqrt{r^2 + [dr/d\theta]^2}\, d\theta$$

About the Line θ = π/2

$$S = 2\pi\int_{\alpha}^{\beta} f(\theta)\cos(\theta)\sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta} f(\theta)\cos(\theta)\sqrt{r^2 + [dr/d\theta]^2}\, d\theta$$