Homogeneous Differential Equations

Homogeneous Functions

- A function $f(x, y)$ is homogeneous if

$$f(tx, ty) = t^n f(x,y)$$

  - ▹ For example, $f(x, y) = x^2 + 2xy$ is homogeneous because

$$f(tx, ty) = (tx)^2 + 2(tx)(ty) = t^2(x^2 + 2xy)$$

  - ▹ The exponent, n, is the *degree* of the homogeneous equation.
    Thus, the example above is a 2nd degree homogeneous equation.

Homogeneous Differential Equations

- A homogeneous differential equation has one of two equivalent forms:

$$M(x,y)dx + N(x,y)dy = 0$$

$$\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$$

  - ▹ $M(x,y)$ and $N(x,y)$ are homogeneous equations of the same degree,

Making a homogeneous differential equation separable

- Use u-substitution, with $u = \frac{y}{x}$
  - ▹ This yields two resulting equations:

$$y = ux$$

$$\frac{dy}{dx} = u + x\,\frac{du}{dx}$$

- Do the substitution back into the original differential equation.
- The result will simplify into a separable differentiable equation in x and u.