

## Area under a curve

### Riemann Approximation

The Riemann approximation of the area under the graph of the function  $f(x)$  on the interval  $[a,b]$  is

*Left approximation:* 
$$\sum_{i=0}^{n-1} f(a + wi) \cdot w$$

*Right approximation:* 
$$\sum_{i=1}^n f(a + wi) \cdot w$$

*Middle approximation:* 
$$\sum_{i=0}^{n-1} f\left(a + \frac{w}{2} + wi\right) \cdot w$$

<b>a, w, n</b>	
$n$	Number of intervals
$a$	Left bound
$w$	Width of interval
	$w = \frac{b-a}{n}$

### Exact Area

The area under the graph of the function  $f(x)$  on the interval  $[a,b]$  is

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(a + \frac{(b-a)i}{n}\right) \cdot \left(\frac{b-a}{n}\right)$$

## Summation Formulas

These are useful in evaluating the exact area limit.

$$\sum_{i=1}^n c = cn$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

$(n+1)(2n+1) = 2n^2 + 3n + 1$   
 You're welcome