Rational Functions' Asymptotes

Holes and Vertical Asymptotes

Factor the numerator and denominator.

$$f(x) = \frac{(x+2)(x-1)}{(x-2)(x-1)}$$

- Elements that cancel out represent holes.

  In our example, there is a hole at $x = 1$.

  Find the y coordinate of the hole by plugging the x value into the reduced version of the function. In our example, the hole is at $(1, -3)$.

- Non-hole values of x that make the denominator zero represent vertical asymptotes.

  In our example, there is a vertical asymptote at $x = 2$.

Horizontal and Slant Asymptotes

The existence and equation for a horizontal or a slant (oblique) asymptote is determined by the relative degrees of the numerator ("top") and denominator ("bottom").

- *Degree top < degree bottom* → ***horizontal*** asymptote at ***y = 0*** (*i.e.*, *x*-axis).

  $$f(x) = \frac{x^2 + 4x - 8}{x^3 - 7x + 2}$$

- *Degree top = degree bottom* → ***horizontal*** asymptote at the ***ratio of the leading coefficients***.

  $$f(x) = \frac{2x^2 + 4x - 8}{3x^2 - 7x + 2} \rightarrow \text{horizontal asymptote at } y = \tfrac{2}{3}$$

- *Degree top = degree bottom + 1* → ***Slant*** asymptote.

  To find the equation of the asymptote, divide the denominator into the numerator, discarding any remainder; the result is the equation of the asymptote.

  $$f(x) = \frac{2x^3 + 4x^2 - 8x + 2}{x^2 - 7x + 2} \rightarrow \text{horizontal asymptote at } y = 2x + 18$$

- *Degree top > degree bottom + 1* → ***No*** horizontal or slant asymptote.

  $$f(x) = \frac{2x^4 + 4x^2 - 8x + 2}{x^2 - 7x + 2} \rightarrow \text{no horizontal asymptote.}$$