Operations

- An *operation* is an action that takes one or two numbers an returns another number.

  For example, addition: $3 + 4 = 7$
  - ▹ The numbers you hand to an operation are the operation's *operands*.
  - ▹ The number it creates is the *result.*
- An abstract operation, say ☺, must define how operands are converted to results.

  a ☺ b = 2a(b − a)

  3 ☺ 5 = 2 × 3 × (5 − 1) = 24

Closure, Commutativity, Associativity

- A set S is *closed* under operation ☺ if, for any two elements of S,

  a ☺ b ∈ S

  e.g., The integers are closed under addition; they are *not* closed under division.
- An operation ☺ is *commutative* if

  a ☺ b = b ☺ a

  Thus, addition is commutative over ℝ, but subtraction is not.

  This is written "ℝ,+"

  Thus, ℝ,℗ where *a* ℗ *b* = a^b is not commutative, because $2^3 \pm 3^2$
- An operation ☺ is *associative* if you can change the grouping of an expression and get the same result, *e.g.,*

  (a ☺ b) ☺ c = a ☺ (b ☺ c)

  Thus, addition is associative: $(5 + 6) + 4 = 5 + (6 + 4)$

  but subtraction is not: $(5 - 6) - 4 \neq 5 - (6 - 4)$

Cayley Tables

- A tabular representation of an operation.
- Commonly used to represent abstract operations.
- For example:

op.	2nd operand			
1st operand				

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

↑	A	B	C	
A	C	B	B	
B	C	A	C	
C	A	C	B	

S = { A, B, C}

S is closed under ↑

S is not commutative under ↑

S is not associative under ↑

(A ↑ A) ↑ A ≠ A ↑ (A ↑ A)

Commutativity

If an operation is commutative, its Cayley table will be symetrical about its upper-left to lower-right diagonal.

Identity and Inverse Elements

Identity Value

- An *identity value* for an operation is a value that leaves the other operand unchanged.

  For example, 0 is the identity value for addition; 1 is the identity value for multiplication.

  In the Cayley table at right, C is the identity element.

↑	A	B	C	D
A	C	B	A	A
B	C	A	B	B
C	A	B	C	D
D	A	C	D	C

Inverse Values

- The *inverse* of an element (symbol: A^{-1}) for a particular opertion is one that combines—in either direction—with the element to yield the identity element for that operation.

  For example, in addition, -6 is the inverse of 6; in multiplication, the inverse value of 3 is ⅓.

  In the Cayley table at right:

  $A^{-1} = A$ (it is its own inverse)

  $B^{-1} = D$

  $C^{-1} = C$ (every identity is its own inverse)

↑	A	B	C	D
A	C	B	A	A
B	C	A	B	C
C	A	B	C	D
D	A	C	D	A