Permutations, Combinations, Probability

Permutations and combinations are the number of ways a particular event can happen: the number of ways you can draw 3 kings in a deck of cards; the number of ways you can assemble a committee from a group of people, etc.

Permutations (Order matters)

- $_nP_r$ (spoken "*n* pick *r*"): How many distinct ways can you arrange *r* items taken from a collection of *n* objects when order matters?

  Other notation: $P(n,r)$ $\quad {}^nP_r$

- *With replacement* (that is, repetition is allowed):

  $_nP_r = n^r$

- *Without replacement* (that is, repetition is *not* allowed):

  The first *r* terms of *n*!, to wit:

  $$_nP_r = \frac{n!}{(n-r)!}$$

You don't play cards?

- There are 52 cards in a deck of cards.
- There are four "suits," (♠♥♣♦) each with 13 cards
- There are 12 "face cards," (Jack, Queen, King).

Distinguishable Permutations

If a set of *n* elements is made up of several sets of repeating elements having lengths n_1, n_2, ..., then the number of distinguishable permutations of the elements is

$$_nP_r = \frac{n!}{n_1! \cdot n_2! \cdot \ldots}$$

e.g., How many ways can you arrange the letters in the word "Mississippi"?

There are 11 letters, consisting of 4 i's, 2 p's, 1 M, and 4 s's

$$\frac{11!}{4! \cdot 2! \cdot 1! \cdot 4!}$$

Combinations (Order Doesn't Matter)

- $_nC_r$ (spoken "*n* choose *r*"): How many distinct ways can you arrange *r* items taken from a collection of *n* objects when order doesn't matter?

  $$_nC_r = \frac{n!}{r!\,(n-r)!}$$

Probability

- The probability of an event *A* happening is

  $$P(A) = \frac{\text{number of } \textit{successful} \text{ outcomes}}{\text{number of } \textit{possible} \text{ outcomes}}$$

  e.g., the probability of rolling a die and getting a 3 is $1/6$

  e.g., the probability of picking a 2 from a deck of cards is $4/52 = 1/13$

Combining Probabilities of Two Events

- If the two events (*A* & *B*) are ***independent,*** then the chance of both happening is

  $$P(A \,\&\, B) = P(A) \times P(B)$$

  e.g., the chance of rolling a die twice and getting a 3 both times is

  $$P(A) \times P(B) = 1/6 \times 1/6 = 1/36$$

- If the two events (*A* & *B*) are ***mutually exclusive,*** then the chance of both happening is

  $$P(A \text{ or } B) = P(A) + P(B)$$

  e.g., the chance of rolling a die twice and getting either a 2 or a 3 is

  $$P(A) + P(B) = 1/6 + 1/6 = 1/3$$

- If the two events (*A* & *B*) are ***not mutually exclusive,*** then the chance of either happening is

  $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

  e.g., the chance of picking a card from a deck and getting either a 2 or a spade

  $$P(A) + P(B) = 1/4 + 1/13 - (1/4 \times 1/13) = 16/52 = 4/13$$

- If the two events (*A* & *B*) are ***conditional,*** then the chance of *A* happening given *B* is

  $$P(A \mid B) = P(A \,\&\, B) / P(B)$$

  "Conditional" means that event *A* can happen only if *B* happens, but *A* doesn't *necessarily* happen if *B* happens.

  e.g., if 90% of the class passed a quiz and 60% of the class passed both the quiz and its following test, what is the probability that a student who passed the quiz also passed the test?

  $$P(A \mid B) = P(A \,\&\, B) / P(B) = .6 / .9 = .667$$